MATHEMATICAL SCIENCES

Programme Code: MATH04

Programme Outcome:

Upon completing the PhD degree in the field of Mathematics, students have/capable of:

- A solid understanding of graduate level algebra, analysis and topology.
- Using their mathematical knowledge to tackle research problems.
- Identifying unsolved yet relevant problems in a specific field.
- Undertaken original research on a particular topic.
- Communicate mathematics accurately and effectively in both written and oral form.
- Conducting scholarly or professional activities in an ethical manner.

DETAILED COURSE STRUCTURE

SEMESTER-I				
Sr.No	**Subject Title**	**Course Code**	**Hours**	**Credits**
1	Algebra - I	MAT601	60	4
2	Analysis - I	MAT603	60	4
3	Topology - I	MAT605	60	4

SEMESTER-II				
Sr.No	**Subject Title**	**Course Code**	**Hours**	**Credits**
1	Complex Analysis	MAT607	60	4
2	Elective - I	MAT***	60	4
3	Elective - II	MAT***	60	4
4	Project	MAT699	60	4
5	RM/RPE	MAT700	60	4

ELECTIVES				
Sr.No	**Subject Title**	**Course Code**	**Hours**	**Credits**
1	Algebra - II	MAT602	60	4
2	Analysis - II	MAT604	60	4
3	Topology - II	MAT606	60	4

Elective – I and Elective – II can be chosen from the above three choices

For the Project, a student may choose a reading project (with a faculty member) or an elective from the offered electives of the school.

CORE COURSES COORDINATOR

Chief Coordinators:

Dr. Tushar Kanta Naik, PGCS Convener, School of Mathematical Sciences
(0674-2494000, Extn-2127, Email: convener_pgcsm@niser.ac.in)

Dr. Panchugopal Bikram, Chairperson, School of Mathematical Sciences
(0674-2494000, Extn-2081, Email: cpsms@niser.ac.in)

Course	Coordinators	Contact
Algebra - I MAT601	Dr. Sudhir Kumar Pujahari Dr. Jaban Meher	0674-2494000, Extn-2094 spujahari@niser.ac.in 0674-2494000, Extn-2097 jaban@niser.ac.in
Analysis - I MAT603	Dr. Manas Ranjan Sahoo Dr. Anupam Pal Choudhury	0674-2494000, Extn-2096 manas@niser.ac.in 0674-2494000, Extn-2218 anupampcmath@niser.ac.in
Topology - I MAT605	Dr. Chitrabhanu Chaudhuri Dr. Panchugopal Bikram	0674-2494000, Extn-2083 chitrabhanu@niser.ac.in 0674-2494000, Extn-2086 bikram@niser.ac.in
Complex Analysis MAT607	Dr. Sudhir Kumar Pujahari Dr. Dinesh Kumar Keshari	0674-2494000, Extn-2094 spujahari@niser.ac.in 0674-2494000, Extn-2124 dinesh@niser.ac.in

ELECTIVES COURSES COORDINATOR

Course	Coordinators	Contact
Algebra - II MAT602	Dr. Krishanu Dan Dr. Senthil Kumar K.	0674-2494000, Extn-2095 krishanu@niser.ac.in 0674-2494000, Extn-2091 senthil@niser.ac.in
Analysis - II MAT604	Dr. Ramesh Manna Dr. Sanjay Parui	0674-2494000, Extn-2099 rameshmanna@niser.ac.in 0674-2494000, Extn-2087 parui@niser.ac.in
Topology - II MAT606	Dr. Chitrabhanu Chaudhuri Dr. Rekha Biswal	0674-2494000, Extn-2083 chitrabhanu@niser.ac.in 0674-2494000, Extn-2466 rekha@niser.ac.in

CORE COURSES

MAT601: Algebra I (60 Lecture Hrs)

Coordinator: Dr. Sudhir Kumar Pujahari, Dr. Jaban Meher
spujahari@niser.ac.in & jaban@niser.ac.in

Course Details:

- Group Theory: Dihedral groups, Permutation groups, Group actions, Sy- low's theorems, Simplicity of the alternating groups, Direct and semidirect products, Solvable groups, Nilpotent groups, Jordan Holder Theorem, free groups.
- Ring Theory: Properties of Ideals, Chinese remainder theorem, Field of fractions, Euclidean domains, Principal ideal domains, Unique factorization domains, Polynomial Rings, Irreducibility criteria, Matrix rings.
- Module Theory: Examples, quotient modules, isomorphism theorems, Generation of modules, free modules, tensor products of modules, Exact sequences - Projective, Injective and Flat modules.

Course Outcomes:

Students will learn basic properties of groups, rings, and modules and will be able to use these algebraic structures to solve research problems.

References:

1. D. S. Dummit and R. M. Foote, Abstract Algebra. John Wiley & Sons, 2004.
2. T. W. Hungerford, Algebra, Graduate Texts in Mathematics, 73, Springer, 1980.
3. M. Artin, Algebra, Prentice Hall, 1991.
4. N. Bourbaki, Algebra, Springer, 1989.
5. C Musili, Introduction to Rings and Modules, Narosa Publishing House.
6. N. S. Gopalakrishnan, University Algebra, New Age International.

MAT603: Analysis I (60 Lecture Hrs)

Coordinator: Dr. Manas Ranjan Sahoo, Dr. Anupam Pal Choudhury
manas@niser.ac.in & anupampcmath@niser.ac.in

Course Details:

- Spaces of functions: Continuous functions on locally compact spaces, Stone- Weierstrass theorems, Ascoli- Arzela Theorem.
- Review of Measure theory: Sigma-algebras, measures, construction and properties of the Lebesgue measure, non-measurable sets, measurable functions and their properties.
- Integration: Lebesgue Integration, various limit theorems, comparison with the Riemann Integral, Functions of bounded variation and absolute continuity.
- Measure spaces: Signed-measures, Radon-Nikodym theorem, Product spaces, Fubini's thoerem (without proof) and its applications.
- Lp-spaces: Holder and Minkowski inequalities, completeness, Convolutions, Approximation by smooth functions.
- Fourier analysis: Fourier Transform, Inverse Fourier transform, Plancherel Theorem for Real numbers.

Course Outcomes:

Upon successful completion of the course, students will be familiar with various advanced concepts and techniques from functional analysis, measure theory and harmonic analysis (on the real line).

References:

1. D. S. Bridges, Foundations of Real and Abstract Analysis, GTM series, Springer Verlag 1997.
2. G. B. Foland, Real Analysis: Modern Techniques and Their Applications (2nd ed.), Wiley-Interscience /John Wiley Sons, Inc., 1999.
3. P. R. Halmos, Measure Theory, Springer-Verlag, 1974.
4. H. L. Royden, Real Analysis, Macmillan 1988.
5. W. Rudin, Real and Complex Analysis, TMH Edition, Second Edition, New-York, 1962.
6. Elliott H. Lieb and Michael Loss , Analysis, American Mathematical Society, 2001.

MAT605: Topology-I (60 Lecture Hrs)

Coordinator: Dr. Chitrabhanu Chaudhuri, Dr. Panchugopal Bikram
chitrabhanu@niser.ac.in & bikram@niser.ac.in

Course Details:

- Topological spaces, Continuous maps between topological spaces, Product topology, Quotient spaces, Connectedness, Compactness, Path connected spaces, Separation axioms, Tychonoff spaces, Urysohn's lemma and Metrization theorem.
- Differentiable functions on Rn, Jacobian criteria, Taylor's theorem, Inverse function theorem, Implicit function theorem, Maxima-minima, Lagrange multiplier.

Course Outcomes:

Upon successful completion of the course, students will aware of various properties of topological space and various properties of functions on topological spaces. The students also learn continuous maps between topological spaces, product topology, Quotient spaces, Connectedness, Compactness, Path connected spaces, separation axioms, Tychonoff spaces, Urysohns lemma and metrization theorem. Furthermore, the student will learn various properties of functions for several variable.

References:

1. Armstrong, Basic Topology, Springer, 1983.
2. Munkres, Topology, Pearson Education, 2005.
3. J. Dugundji, Topology.
4. J. J. Duistermaat, J. A. C. Kolk: Multidimensional Real Analysis I: Differentiation.
5. K. Janich, Topology, Springer.
6. John L Kelley: General Topology (free download: https://archiveorg/details/GeneralTopology)
7. F. Simmons: Introduction to Topology and Modern Analysis.
8. S.Kumaresan: A Course in Differential Geometry and Lie Groups, TRIM series.
9. T. M. Apostol: Calculus: Multi-Variable Calculus and Linear Algebra With Applications to Differential Equations And Probability- Vol 2.

MAT607: Complex Analysis (60 Lecture Hrs)

Coordinator: Dr. Sudhir Kumar Pujahari, Dr. Dinesh Kumar Keshari
spujahari@niser.ac.in & dinesh@niser.ac.in

Course Details:

Review of basic Complex Analysis: Cauchy-Riemann equations, Cauchy's theorem and estimates, power series expansions, maximum modulus principle, Classification of singularities and calculus of residues; Normal families, Arzela-Ascoli theorem, Riemann mapping theorem; Weierstrass factorization theorem, Runges theorem, Mittag-Lefiers theorem; Hadamard factorization theorem, Analytic Continuation, Gamma and Zeta functions.

Course Outcomes:

Upon successful completion of the course, students will learn some important theorems in complex analysis such as Riemann mapping theorem, Weierstrass factorization theorem, Runges theorem, Hardamard factorization theorem, Little Picards theorem and Great Picards theorem. They will also learn some basic techniques of harmonic functions and characterization of Dirichlet Region. These results are very useful in many branches of mathematics such as Number Theory, Differential Geometry, Operator theory, Partial Differential Equations etc.

References:

1. L. V. Ahlfors, Complex Analysis, Tata McGraw-Hill, 2013.
2. J. B. Conway, Functions of one complex variable, Second edition. Graduate Texts in Mathematics, 11. Springer-Verlag, New York-Berlin, 1978.
3. R. Narasimhan and Y. Nievergelt, Complex analysis in one variable, Second edition. Birkhuser Boston, Inc., Boston, MA, 2001.
4. W. Rudin, Real and Complex Analysis, Tata McGraw-Hill, 2013.
5. Wolfgang Fischer, Ingo Lieb, A Course in Complex Analysis: From Basic Results to Advanced Topics, Springer, 2012
6. Eberhard Freitag, Rolf Busam, Complex Analysis, Springer, 2005.
7. Stein and Shakarchi, Complex Analysis, Princeton University Press, 2003.
8. T. Gamelin, Complex Analysis, Springer, 2000

ELECTIVE COURSES

MAT602: Algebra II (60 Lecture Hrs)

Coordinator: Dr. Krishanu Dan, Dr. Senthil Kumar K.

krishanu@niser.ac.in & senthil@niser.ac.in

Course Details:

- Linear Algebra: Matrix of a Linear transformation, dual vector spaces, determinants, Tensor algebras, Symmetric algebras, Exterior algebras,
- Modules over PIDs: Basic theory, Structure theorem for finitely generated abelian groups, Rational and Jordan canonical forms.
- Field Theory: Algebraic extensions, Splitting fields, Algebraic closures, Sep- arable and Inseparable extensions, Cyclotomic polynomials and extensions, Galois extensions, Fundamental Theorem of Galois theory, Finite fields, Composite extensions, Simple extensions, Cyclotomic extensions and Abelian extensions over rational field, Galois groups of polynomials, Fundamental theorem of algebra, Solvable and Radical extensions, Computation of Galois groups over rational field.

Course Outcomes:

Students will learn basic properties of fields and Galois theory and will be able to use these results to solve other mathematical problems.

References:

1. D. S. Dummit and R. M. Foote, Abstract Algebra. John Wiley & Sons, 2004.
2. T. W. Hungerford, Algebra, Graduate Texts in Mathematics, 73, Springer, 1980.
3. M. Artin, Algebra, Prentice Hall, 1991.
4. T. T. Moh: Algebra, World Scientific, 1992
5. N. Bourbaki, Algebra, Springer, 1989.

MAT604: Analysis II (60 Lecture Hrs)

Coordinator: Dr. Ramesh Manna, Dr. Sanjay Parui
rameshmanna@niser.ac.in & parui@niser.ac.in

Course Details:

Banach spaces: Review of Banach spaces, Hahn-Banach Theorem and its applications, Baire Category theorem and its applications like Closed graph theorem, Open mapping theorem. Topological Vector spaces: Weak and weak* topologies, locally convex topological vector spaces. Hilbert spaces: Review of Hilbert spaces and operator Theory, Compact operators, Schauder's theorem on the spectral theory of compact operators. Banach algebras: Elementary properties, Resolvent and spectrum, Spectral radius formula, Ideals and homomorphisms, Gelfand transforms, Gelfand theorem for commutative Banach algebras.

Course Outcomes:

Upon successful completion of the course, students will aware of various properties of norm linear vector spaces and topological vector spaces. They will also learn various properties of linear transformations defined on these norm liner spaces and topological vector spaces.

References:

1. D. S. Bridges, Foundations of Real and Abstract Analysis, GTM series, Springer Verlag 1997.
2. G. B. Foland, Real Analysis: Modern Techniques and Their Applications (2nd ed.), Wiley-Interscience /John Wiley Sons, Inc., 1999.
3. G. K. Pederson, Analysis NOW, GTM series, Springer-Verlag, 1991.
4. W. Rudin, Real and Complex Analysis, TMH Edition, Second Edition, New-York, 1962.
5. W. Rudin, Functional Analysis, TMH Edition, 1974.
6. K. Yosida, Functional Analysis, Springer-Verlag 1968.

MAT606: Topology II (60 Lecture Hrs)

Coordinator: Dr. Chitrabhanu Chaudhuri, Dr. Rekha Biswal
chitrabhanu@niser.ac.in & rekha@niser.ac.in

Course Details:

- Homotopy Theory: Fundamental groups and its functorial properties, examples, Van-Kampen Theorem, Computation of fundamental group of S1.
- Covering spaces: Covering spaces, Computation of fundamental groups using coverings. The classification of covering spaces. Deck transformations.
- Simply connected spaces: Simply connected spaces-Universal covering spaces of locally simply connected and pathwise connected spaces. Universal covering group of connected subgroups of General Linear groups.
- Homology groups: Affine spaces, simplexes and chains - Homology groups - Properties of Homology groups. Chain Complexes, Relation Between one dimensional Homotopy and Homology groups. Computation of Homology groups Sn, Brouwer's fixed point theorem.

Course Outcomes:

Upon successful completion of the course, students will learn covering spaces, homotopy theory and several concepts of topological invariants, namely, fundamental groups, homology groups.

References:

1. M. A. Armstrong, *Basic Topology*, Springer, 1983
2. M. J. Greenberg & J. R. Harper, *Algebraic Topology*: A First Course, Addition Wesley, 1984.
3. J. R. Munkres, *Topology*, Pearson Education, 2005. 1974